Hidden Markov Occupancy Modelling

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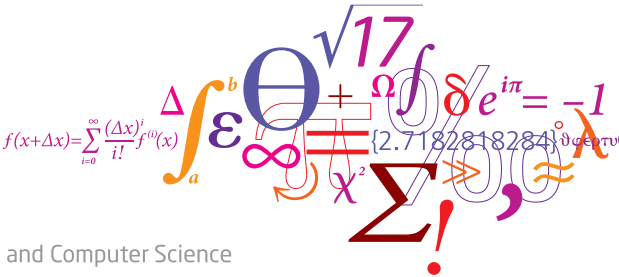
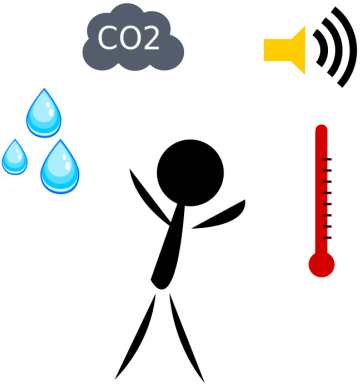
Dynamical Systems, DTU Compute



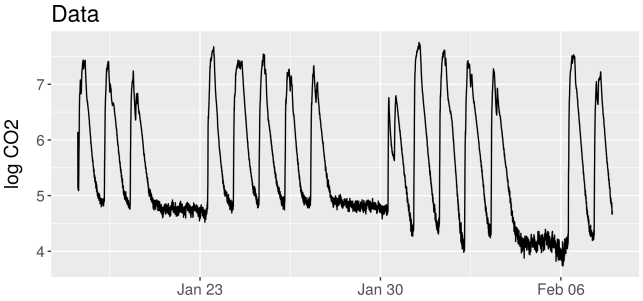
DTU Compute
Department of Applied Mathematics and Computer Science

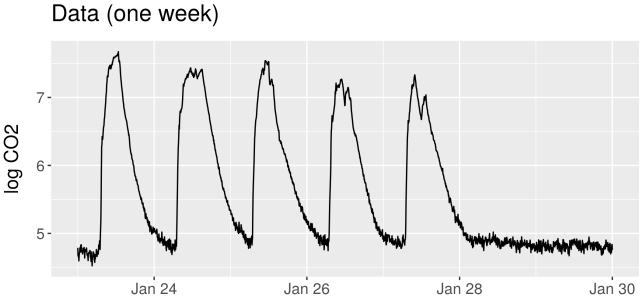
Modelling people's presence in buildings

- improve HVAC control using information about occupants' presence
- input for building simulations
- basis for modelling of other behaviour (windows, heating,...)
- presence often not directly measurable $\Rightarrow$ indirect methods

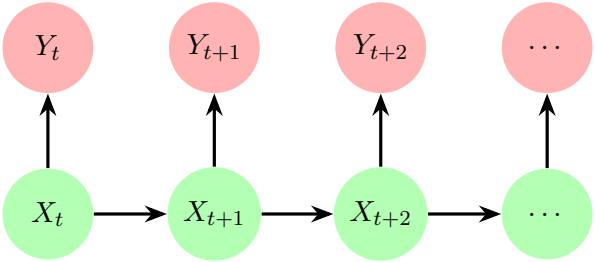


Data


Data
log CO2
7
6
5
4
Jan 23
Jan 30
Feb 06


Data (one week)
log CO2
7
6
5
Jan 24
Jan 26
Jan 28
Jan 30

Hidden Markov Model - homogeneous



The model can be expressed by

$$p\left(X_t = i \mid X_{t-1} = j\right) \sim (\Gamma)_{i,j}$$
$$Y_t = c_i + \varepsilon_{i,t}$$

where $\Gamma \in \mathbb{R}^{m \times m}$ is a transition probability matrix, c_i are the state means and $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$.

Viterbi Algorithm

Calculation of the most likely sequence of states $i_1^*, i_2^*, ..., i_T^*$ (global decoding). For

$$\xi_{ti} = \max_{x_1, x_2, \cdots, x_{t-1}} P(\mathbf{X}^{(t-1)} = \mathbf{x}^{(t-1)}, X_t = i, \mathbf{Y}^{(t)} = \mathbf{y}^{(t)}))$$

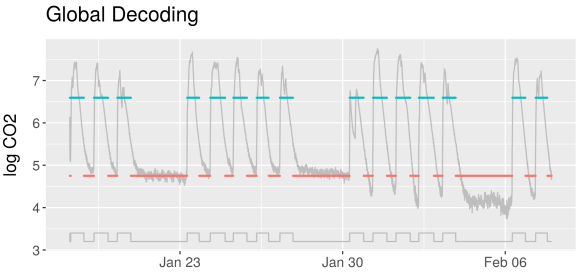
the recursion holds:

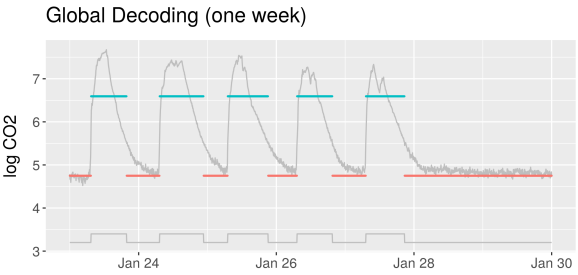
$$\xi_{tj} = \max_i (\xi_{t-1,i} \Gamma_{ij}) p_j(x_t)$$

$$i_t^* = \arg\max_{i=1,\ldots,m} (\xi_{ti} \Gamma_{i,i+1})$$

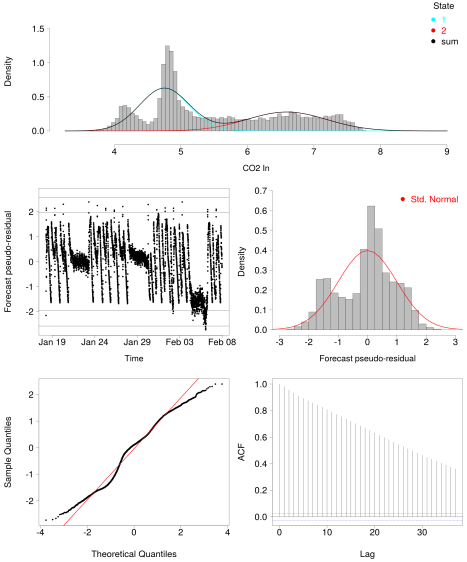
- Brute force maximization: $\mathcal{O}(m^T)$ calculations
- Viterbi algorithm $\mathcal{O}(T)$ calculations.

2-state Hidden Markov Model

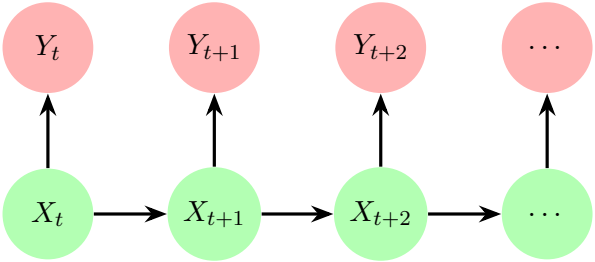




2-state Hidden Markov Model



Hidden Markov Model - inhomogeneous



The model can be expressed by

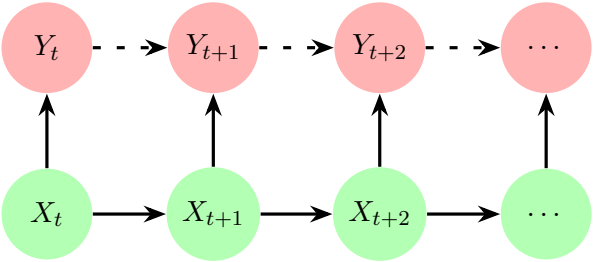
$$p\left(X_t = i \mid X_{t-1} = j\right) \sim \left(\Gamma_t\right)_{i,j}$$
$$Y_t = c_i + \varepsilon_{i,t}$$

where $\underline{\Gamma_t \in \mathbb{R}^{m \times m}}$ is a <u>inhomogeneous</u> transition probability matrix, c_i are the state means and $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$.

Comparison of homogeneous and inhomogeneous Hidden Markov Model

m=2 states	parameters	AIC	BIC
homogeneous	6 $(m^2 + m)$	7792	7832
inhomogeneous	14 $(m^2 + 5m)$	7627	7721

Markov-Switching AR(1) Model

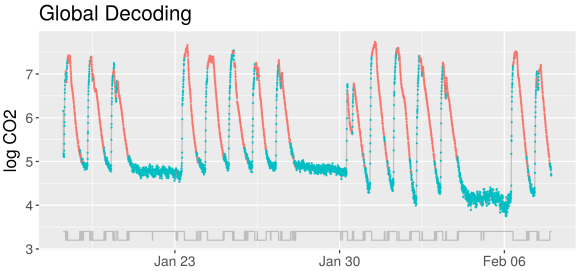


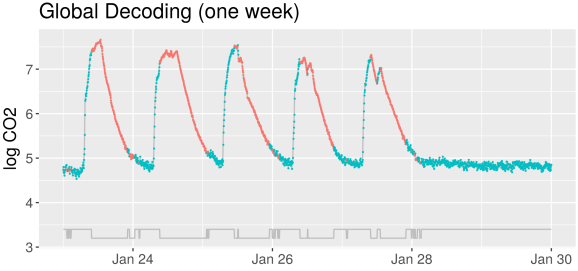
The model can be expressed by

$$p\left(X_t = i \mid X_{t-1} = j\right) \sim \left(\Gamma_t\right)_{i,j}$$
$$Y_t = \underline{\phi_i Y_{t-1}} + c_i + \varepsilon_{i,t}$$

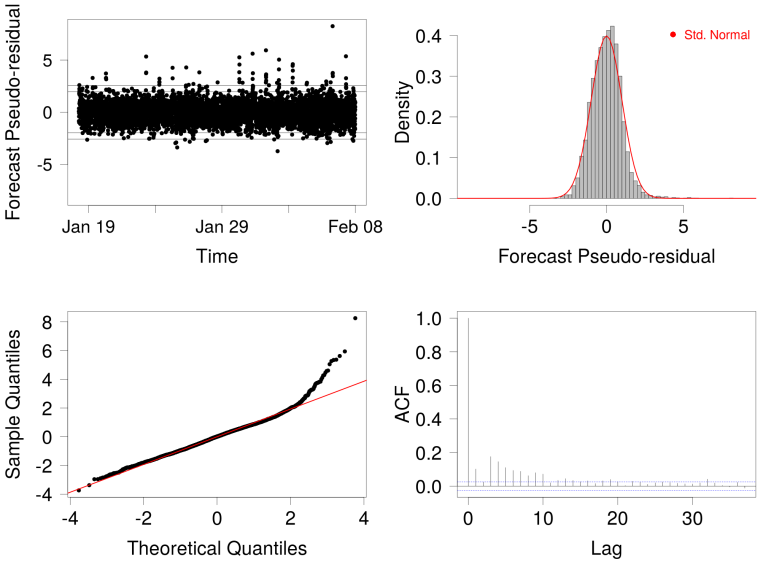
where $\Gamma_t \in \mathbb{R}^{m \times m}$ is a transition probability matrix, c_i are the state means, ϕ_i the auto-regressive parameters and $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$.

2-state Markov-switching





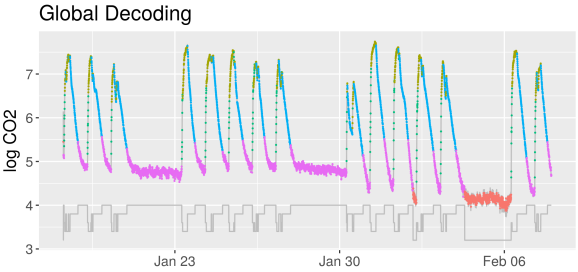
2-state Markov-switching

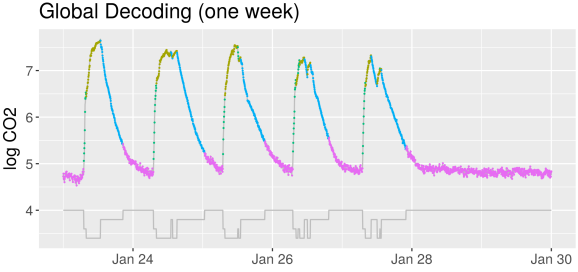


Comparison of Markov-switching Models

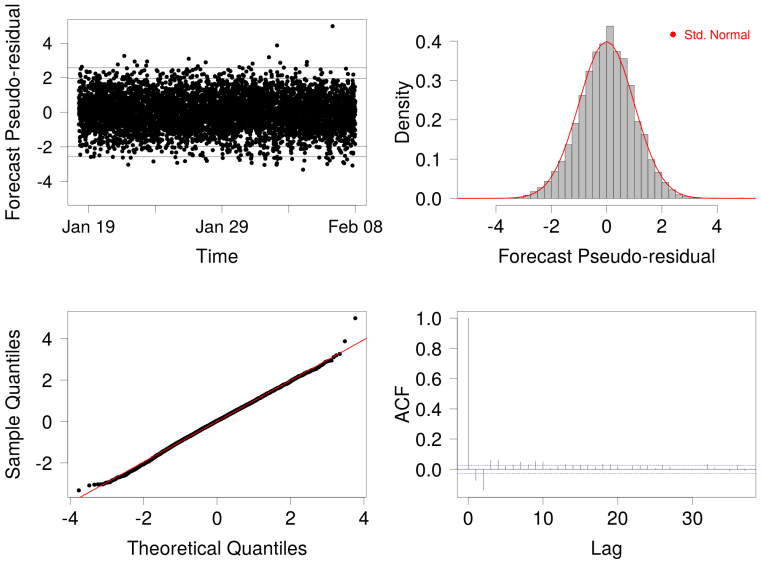
states	parameters	AIC	BIC
2	16	-19187	-19080
3	27	-20326	-20146
4	40	-20771	-20503
5	55	-21392	-21023

5-state Markov-switching

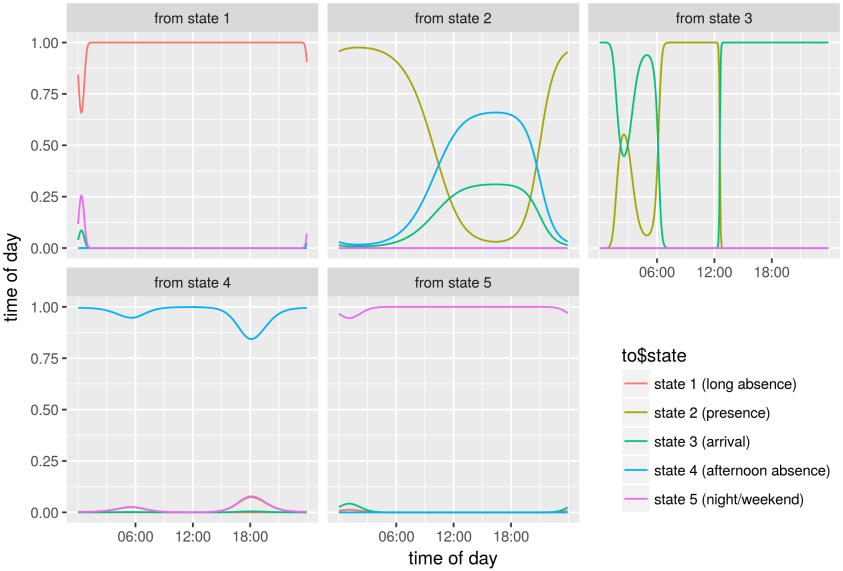




5-state Markov-switching



Transition Probabilities



Outlook

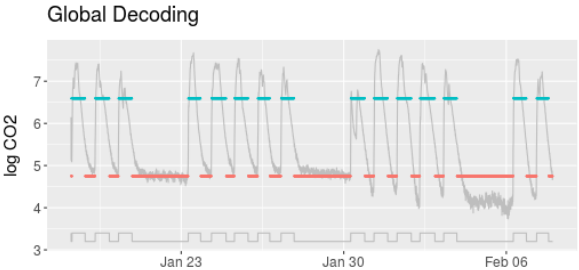
- ground truth validation
- further input variables
  - temperature
  - noise
  - humidity

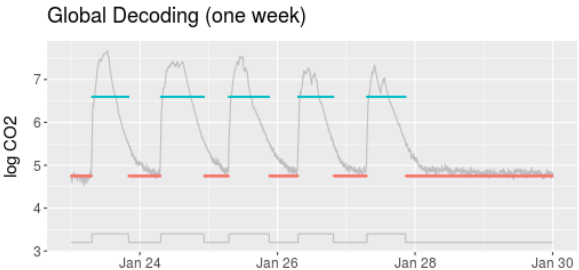
Thank you

sewo@dtu.dk

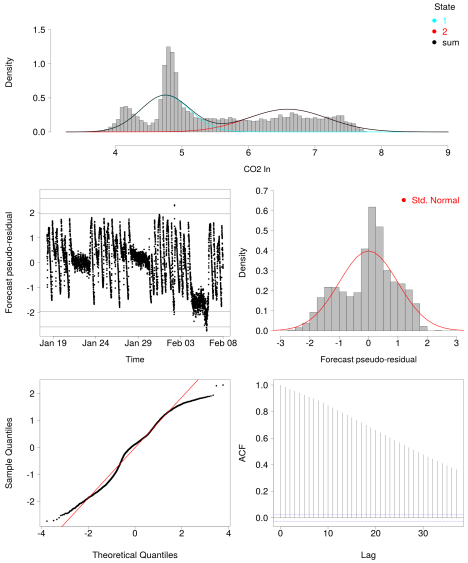
Backup Slides - Hidden Markov Model

2-state Hidden Markov Model

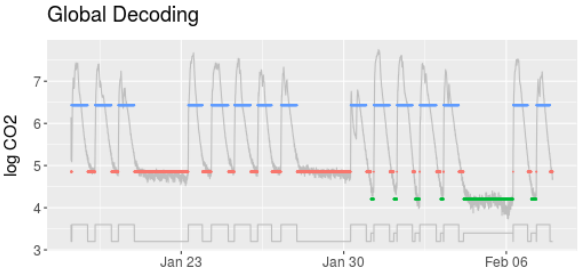


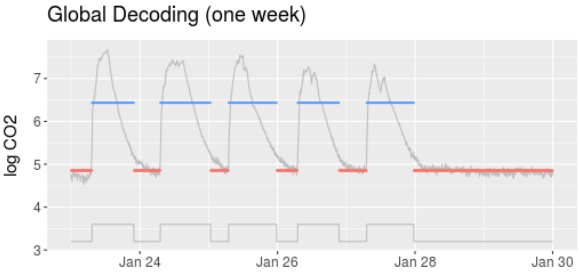


2-state Hidden Markov Model

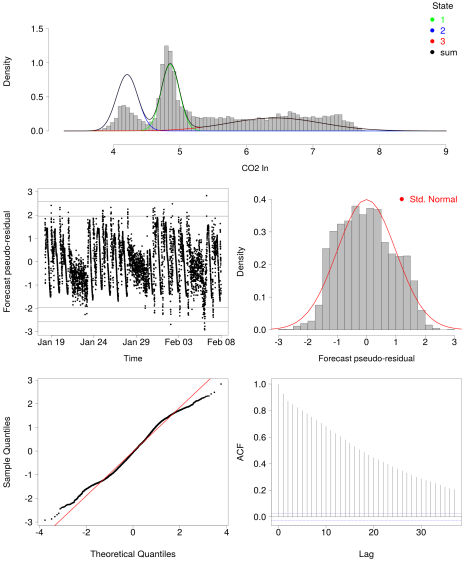


3-state Hidden Markov Model

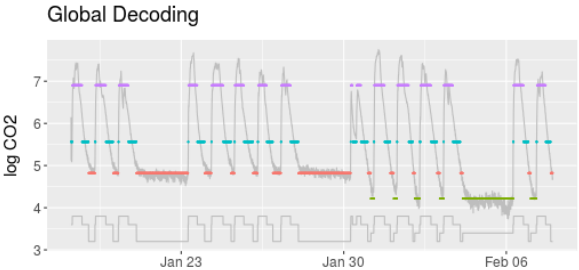


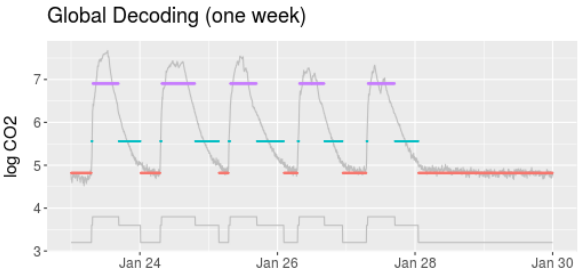


3-state Hidden Markov Model

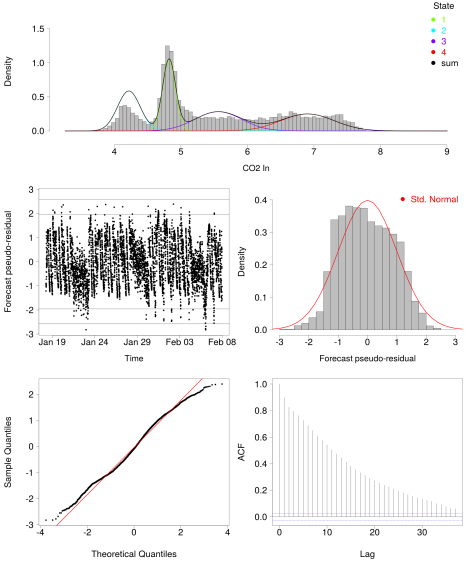


4-state Hidden Markov Model

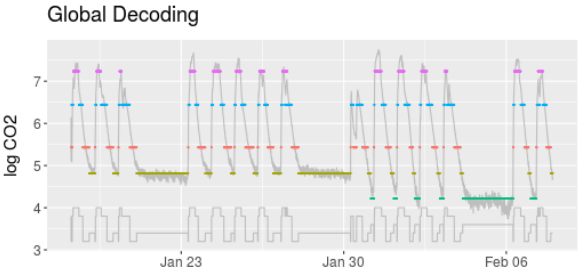


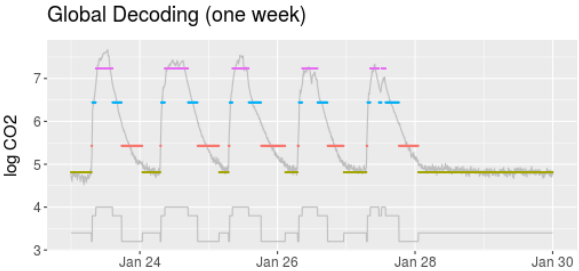


4-state Hidden Markov Model



5-state Hidden Markov Model





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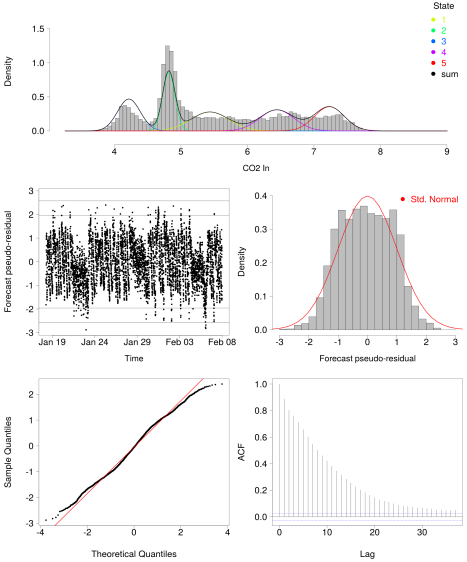
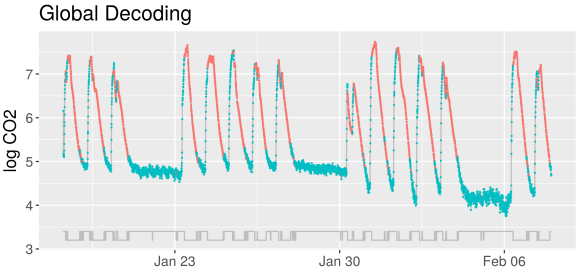
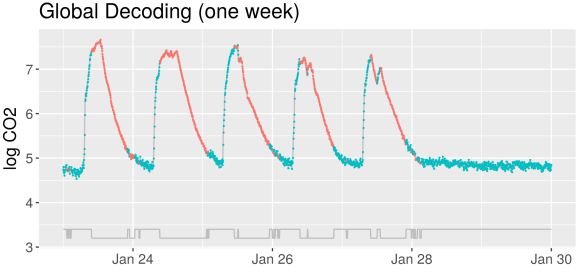


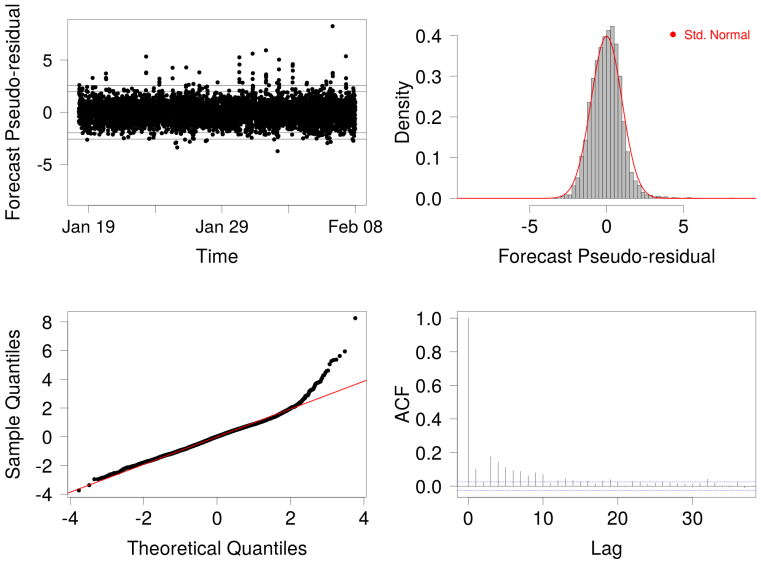
Backup Slides - Markov-switching Model

2-state Markov-switching





2-state Markov-switching



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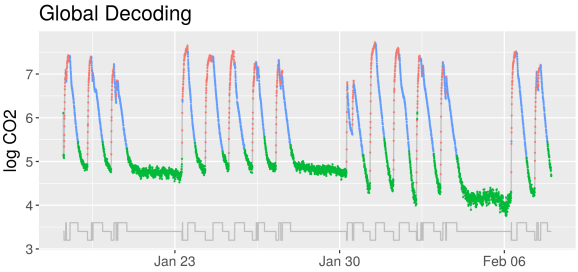

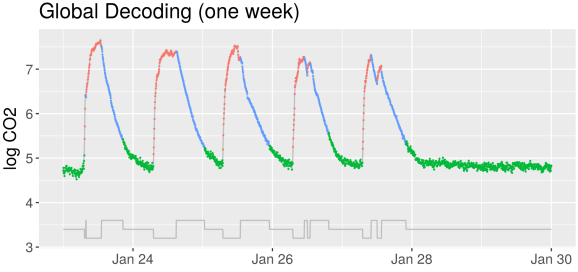

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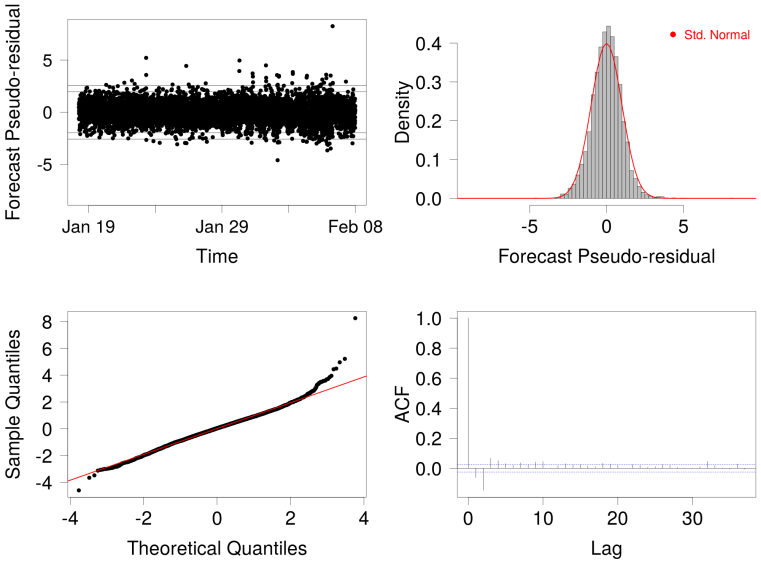
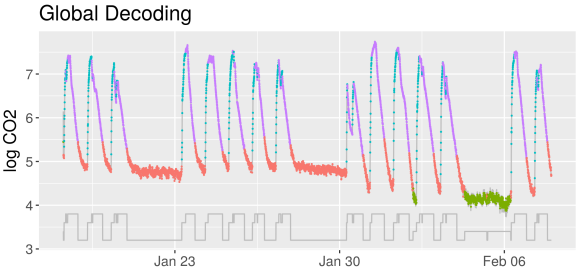
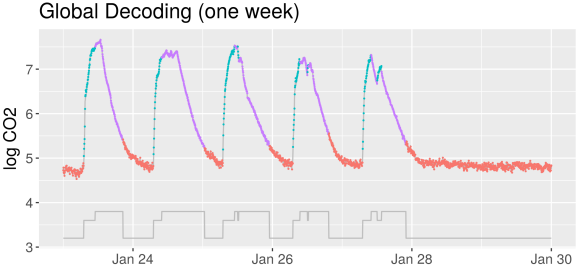


4-state Markov-switching





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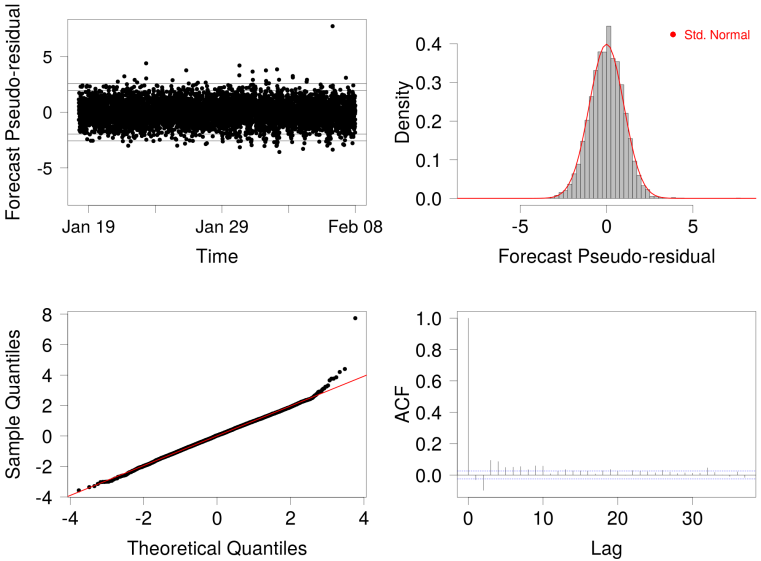

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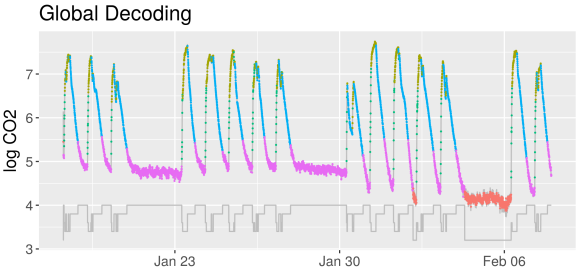

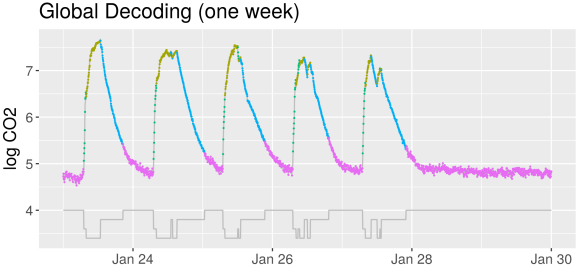

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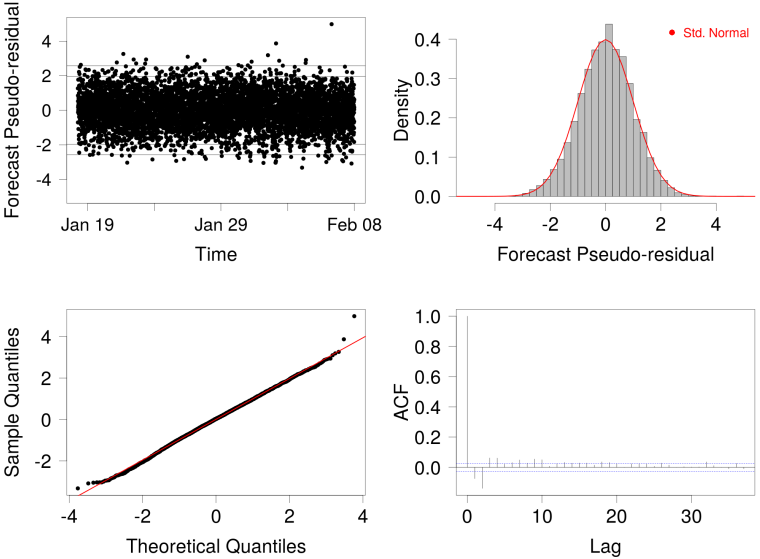

Forecast pseudo-residuals

$$z_t = \Phi^{-1}\left(P(Y_t \leq y_t | \mathbf{Y}^{(t-1)} = \mathbf{y}^{(t-1)})\right),$$

where Φ is the distribution function of the standard normal distribution.